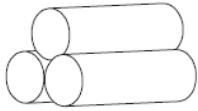


# TAPS 2011

## Round 4

16. Three cylinders are placed on a table, as shown in the diagram below. Each cylinder has a diameter of 150 centimeters. How tall is the stack of cylinders?

Round to the nearest cm. (12-03-4)



17. Five years ago, I planted a four foot-tall tree in my front yard. It has grown the same amount each year, and last year it was twice as tall as it was at the end of the first year. How tall is the tree now? (12-03-9)



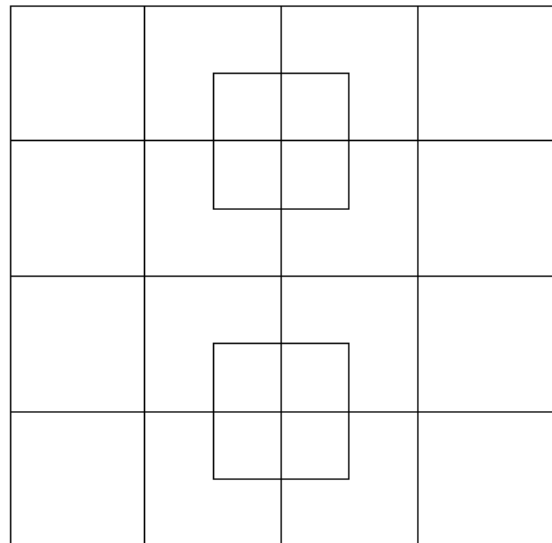
18. During the Christmas holiday season, many postal workers have to work overtime, so a supervisor at one large post office planned a late-night snack for the employees. She ordered 1 extralarge pizza for every two workers, 1 large bag of potato chips for every three workers, and 1 two-liter bottle of cola for every four workers. When the order arrived, 26 items were delivered. How many employees were working that evening? (12-03-5)

19. Five different playing cards are in a pile, face down.

Using the following information, identify the suits and values of the cards from the top to the bottom of the pile. (12-03-7)

- The ace is nearer the top than either the jack or the queen.
- The diamond is nearer the top than either the spade or the club.
- The 10 is neither a heart nor a club and is just above the ace.
- The jack is neither a heart nor a spade and is just below the queen.
- The king is neither a heart nor a spade and is higher than the ace.
- One heart is just above another heart.

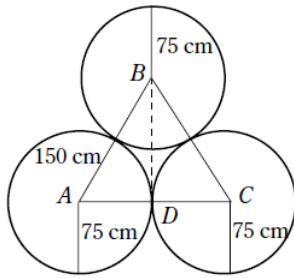
20. How many complete squares are in this figure? (12-03-11)



# TAPS 2011

## Round 4

16.  $(150 + 75\sqrt{3}) = 280$  cm.



If you join the centers of the cylinders, you obtain an equilateral triangle,  $ABC$ , with sides equal to 150 cm. The height,  $BD$ , is the long side of the right triangle,  $ABD$ , whose length is  $\sqrt{3}$  times the length of the short side:  $BD = 75\sqrt{3}$  cm. To length  $BD$  we must add the 75 cm below and 75 cm above the triangle to get the total height of the stack.

17. 14 feet.

Let  $x$  feet be the growth each year. At the end of the first year, the tree was  $4 + x$  ft. tall and at the end of the fourth year it was  $4 + 4x$  ft. tall. Since the height at the end of the fourth year was twice the height at the end of the first year, we can set up this equation:  $4 + 4x = 2(4 + x)$ . Solving this equation, we find that  $x = 2$  ft. Since the tree grows 2 feet each year, it has grown 10 feet in 5 years, and is now 14 feet tall.

18. 24 workers.

Suppose there are  $x$  workers. The number of pizzas delivered will be  $(x \text{ workers}) / (2 \text{ workers/pizza})$  and similarly for the other two items. The total number of items is then  $x/2 + x/3 + x/4 = 26$ . Solving for  $x$ , we find that there must be 24 workers that evening. We could also reason that the number of workers must be a common multiple of 2, 3, and 4. These multiples are 12, 24, 36, 48, 60, 72 . . . . Set up a ratio table, giving the number of items for each possible number of workers, to find that 24 workers result in the given numbers of items

19.  $K\spadesuit, 10\clubsuit, A\heartsuit, Q\heartsuit, J\clubsuit$ .

Clue 2 tells us that one card is a diamond, one is a spade, and one is a club. Clue 6 tells us that there are two hearts and that they are together. The clues also tell us that the cards are an ace, 10, jack, queen, and king. Thus, all the cards are accounted for, and we need to match suits and values. The clues tell

us that the 10, jack, and king are not hearts; therefore, two cards must be the ace of hearts and the queen of hearts and that these two must be together in the pile. Drawing pictures representing the information in the clues is a good approach to this problem. Since the jack is just below the queen, the king is higher than the ace, and the 10 is just above the ace, the order of the king, 10, hearts, and jack has to be  $(K\clubsuit \text{ or } K\spadesuit)(10\clubsuit \text{ or } 10\spadesuit), A\heartsuit, Q\heartsuit, (J\clubsuit \text{ or } J\spadesuit)$ . The king, 10, and jack are shown as both possibilities allowed by the clues. Since there is only one diamond in the pile, the 10 must be the spade, and we have  $(K\clubsuit \text{ or } K\spadesuit), 10\spadesuit, A\heartsuit, Q\heartsuit, (J\clubsuit \text{ or } J\spadesuit)$ . The king and jack are shown in opposite order in the parentheses, because if the king is a club, then the jack has to be a diamond, and vice versa. But we know that the diamond is nearer the top than either the club or the spade, so the king must be the diamond and we have  $K\spadesuit, 10\spadesuit, A\heartsuit, Q\heartsuit, J\clubsuit$ .

20. 40.

From this variation of the classic "checkerboard problem," we get the following:

$$\begin{aligned} 1 \cdot 1 \text{ squares} &\rightarrow 4 \cdot 4 = 16 \text{ squares} \\ 2 \cdot 2 \text{ squares} &\rightarrow 3 \cdot 3 = 9 \text{ squares} \\ 3 \cdot 3 \text{ squares} &\rightarrow 2 \cdot 2 = 4 \text{ squares} \\ 4 \cdot 4 \text{ square} &\rightarrow 1 \cdot 1 = 1 \text{ square} \end{aligned}$$

In addition to these 30 checkerboard squares, each of the "middle" squares results in 4 small  $(1/2 \cdot 1/2)$  squares and 1 large  $(1 \cdot 1)$  square, for a total of 5. Double the 5 and get 10 for the "middle" squares, for a total of 40 squares altogether.